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Compensating fields, bosonization and soldering in QCD_2

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Abstract. An interesting tool for investigating the quantum features of a field theory is the introduction of compensating fields. For instance, the anomalous divergence of the chiral current can be calculated in the field–antifield formalism from an extended form of quantum chromodynamics (QCD) with compensating fields. The interpretation of this procedure from the bosonized point of view, in the two-dimensional case, depends crucially on the possibility of defining a bosonized version for the extended theory. We show, by using some recent results on the soldering of bosonized actions corresponding to chiral fermions, the mapping between bosonic and fermionic representations of this extended QCD_2. In the bosonic formulation the anomalous divergence of the chiral current shows up from the equations of motion of the compensating fields.

1. Introduction

Compensating fields can be defined as those that enlarge the local symmetry content of an action in such a way that the original theory is recovered at the unitary gauge. This means that classically the number of degrees of freedom is not changed by the introduction of a set of compensating fields as they can be completely removed by the gauge-fixing procedure. This kind of field space enlargement has been used in a large number of works in the context of Hamiltonian [1] as well as Lagrangian [2, 3] descriptions of field theories. A fundamental question arises when we introduce compensating fields: are the corresponding new symmetries obstructed by quantum effects? Several examples, within a Lagrangian formalism, show that compensating fields do not add new anomalies [4–6] (in other words, they do not change the BRST cohomology [7]). It can be shown that this is also the case within Hamiltonian descriptions [8]. In spite of possible cohomologically trivial contributions to a field theory, compensating fields are an important tool for extracting information about its quantum features. Recent examples are those of [5, 6] where a general procedure for calculating anomalous divergences of global currents in the field–antifield framework was developed. In these articles, the introduction of compensating fields leads to quantum corrections to the master equation and therefore to additional terms in the quantum action. These quantum corrections make it possible to extract the anomalous divergences of Noether currents from the sole imposition of independence of the vacuum functional with respect to the gauge fixing. These results were illustrated with quantum chromodynamics (QCD) in four dimensions.

In the present work we would like to explore some consequences of the introduction of compensating fields in QCD in two spacetime dimensions†. As is well known, field theories

† For a review on QCD_2 see [9].

defined in two dimensions have an interesting particular aspect: a two-dimensional gauge theory with fermionic matter fields can be mapped into a corresponding theory involving only bosonic fields. The structure and the transformation properties of the bosonized version of a fermionic gauge theory depend essentially on the form of the coupling to the gauge field. For example, in standard QCD_2, where the gauge field is coupled to the vector matter current, the corresponding bosonic matter field is gauge invariant. On the other hand, in the chiral QCD_2 case, where the gauge field is coupled to the chiral current, the bosonic matter field transforms as an element of the gauge group. Therefore, if one tries to look at the introduction of compensating fields as in [5, 6] from the bosonized point of view, one faces the problem of how to find the appropriate bosonized version of the theory. Considering the case of a pure vector coupling, like QCD_2, the corresponding bosonized version is well known. However, the introduction of compensating fields corresponds to coupling an additional pure gauge field to just one of the chiralities. So, the theory including compensating fields cannot be bosonized in the same way. We will see that this problem can be solved by generalizing recent results from [10] on how to solder [11] two chiral fermions, but here with different gauge fields in each chiral sector.

We will see that when we carefully include in the bosonic formulation the counterterms that solve the quantum master equation at one-loop order for the fermionic case, we arrive at a bosonized action where the compensating fields play the role of collective fields as in [3]. It is interesting to emphasize that it is just these counterterms that make it possible to have the complete set of symmetries manifest in the bosonized version of the theory. The introduction of the counterterms also allows us to derive a complete mapping between the chiral currents and their anomalous divergences in the two formulations of the model. It is natural then to interpret some of the equations of motion in the gauged WZW model [9, 12] as being just the bosonic form of the anomalous divergence of the Noether chiral currents.

This work is organized as follows. In section 2 we briefly review the ideas of [5, 6] and present the corresponding results for the case of QCD_2. In section 3 the Abelian version of the model discussed in section 2 is bosonized using the soldering techniques. A mapping between the bosonic and fermionic descriptions of the currents and their divergences is also presented. Section 4 essentially generalizes the results derived in section 3 to the non-Abelian situation. We devote section 5 to some general comments and concluding remarks.

2. Compensating fields and the gauged chiral symmetry in QCD_2

In [6] we have shown that any group G of rigid internal transformations of a set of fields ϕ^i, $i = 1, 2, \ldots, n$, can be used to enlarge the local symmetry content of an action $S_0[\phi^i]$. Local symmetries are introduced when the group elements are promoted to compensating fields in a proper way. To show how this works, let us consider that the action of $g \in G$ over ϕ^i is

$$\phi^{i'} = \phi^{i'}(\phi, g) \tag{2.1}$$

where a subsequent transformation of $\phi^{i'}(\phi, g)$ under a group element h can be obtained directly from ϕ^i through the action of hg. The usual group axioms are assumed, and the identity transformation is generated by the unity element **1** of G. According to (2.1),

$$\phi^i = \phi^i\big(\phi', g^{-1}\big). \tag{2.2}$$

Writing $S_0[\phi^i]$ with the aid of (2.2) and dropping the primes enables us to obtain an action $S_1 = S_1[\phi^i, g(x)]$ with a local set of symmetries which comes from a convenient left multiplication of the group elements, now taken as local fields. Applying these ideas to the Yang–Mills action in two dimensions with G as the group of non-Abelian chiral transformations, $S_1[\phi^i, g(x)]$ can be written as

$$S_1[\psi, \overline{\psi}, A_\mu, g] = \int \mathrm{d}^2x \left(-\tfrac{1}{4}\,\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \mathrm{i}\overline{\psi}\gamma^\mu\big(\partial_\mu - \mathrm{i}\tilde{A}_\mu\big)\psi\right) \tag{2.3}$$

where

$$\tilde{A}_\mu = \tilde{A}_\mu[A_\mu, g] = P_- A_\mu + P_+ B_\mu. \tag{2.4}$$

In the above expressions, $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ are the chiral projectors and the composite field

$$B_\mu = g^{-1}A_\mu g + \mathrm{i}g^{-1}\partial_\mu g \tag{2.5}$$

corresponds to a finite gauge transformation of the Yang–Mills connection A_μ. We can see that $S_1[\psi, \overline{\psi}, A_\mu, g]$ is invariant under the set of local gauge transformations

$$\begin{aligned} \delta\psi &= \mathrm{i}(\eta(x) - \epsilon(x)P_+)\psi & \delta\overline{\psi} &= -\mathrm{i}\overline{\psi}(\eta(x) - \epsilon(x)P_-) \\ \delta A_\mu &= \partial_\mu\eta(x) + \mathrm{i}[\eta(x), A_\mu] & \delta g &= \mathrm{i}(g\epsilon(x) + [\eta(x), g] \end{aligned} \tag{2.6}$$

where $\epsilon = \epsilon^a T^a$, $\eta = \eta^a T^a$ take values in the $SU(N)$ algebra, with generators T^a satisfying $[T^a, T^b] = \mathrm{i}f^{abc}T^c$, $\mathrm{tr}(T^aT^b) = \delta^{ab}$. We assume here that the connections and the fermions belong to the fundamental representation of $SU(N)$.

Transformations (2.6) close in an algebra: $[\delta_1, \delta_2]\phi^i = \delta_3\phi^i$ for any field $\phi^i = \{\psi, \bar{\psi}, A_\mu, g\}$ when the composition rules for the parameters of the transformation are given by

$$\begin{aligned} \eta_3 &= \mathrm{i}[\eta_1, \eta_2] \\ \epsilon_3 &= \mathrm{i}\big([\eta_1, \epsilon_2] + [\epsilon_1, \eta_2] - [\epsilon_1, \epsilon_2]\big) \end{aligned} \tag{2.7}$$

which shows the semi-direct product character of $SU(N) \times SU(N)$ for the gauge structure found in (2.6).

In the gauge $g = \mathbf{1}$, S_1 trivially reduces to the usual Yang–Mills action and the local chiral symmetry is no longer manifest. Furthermore, for $g = \mathbf{1} + \mathrm{i}\beta$, we obtain

$$\left.\frac{\delta S_1}{\delta\beta^a}\right|_{\beta=0} = \big(D_\mu J_R^\mu\big)^a \tag{2.8}$$

which was the starting point, in [6], for deriving the anomalous divergence of the chiral current $J_R^{\mu a} = \overline{\psi}\gamma^\mu T^a P_+\psi$. In (2.8),

$$\big(D_\mu J_R^\mu\big)^a \equiv \partial_\mu J_R^{\mu a} + f^{abc}A_\mu^b J_R^{\mu c}. \tag{2.9}$$

The quantization of such a theory, along the field–antifield formalism†, starts by constructing the BV action

$$\begin{aligned} S = S_1 + \int \mathrm{d}^2x\, \big(&\mathrm{i}\psi^*(c - bP_+)\psi - \mathrm{i}\overline{\psi}(c - bP_-)\overline{\psi}^* + \mathrm{Tr}\{\mathrm{i}g^*(gb + [c, g]) \\ &+ A_\mu^* D^\mu c + \tfrac{1}{2}\mathrm{i}c^*[c, c] - \tfrac{1}{2}\mathrm{i}b^*([b, b] - 2[c, b])\}\big) \end{aligned} \tag{2.10}$$

where the set of fields $A_\mu, g, \bar{\psi}$ and ψ has been extended to include ghosts c and b (corresponding to parameters η and ϵ, respectively) and also antifields (sources of BRST variations) associated with each field. In (2.10) the brackets represent graded commutators.

† For a review of field antifield quantization see [7, 13].

As is well known, the BRST variation of any functional $X(\phi, \phi^*)$ is given by

$$sX = (X, S) \tag{2.11}$$

with S given by (2.10) and the antibrackets defined in such a way that for any two local functionals of the fields and antifields X and Y, $(X, Y) = (\partial^R X/\partial\Phi^A)(\partial^L Y/\partial\Phi^*_A) - (\partial^R X/\partial\Phi^A_*)(\partial^L Y/\partial\Phi_A)$. In the above equation and when pertinent, we are using the de Witt notation of sum over repeated indices and integration over the corresponding intermediary variables. By construction, S is BRST invariant, but the functional generator with S as the quantum action will be well defined at one-loop order only if $\Delta S = 0$ or if S can be extended to some $W = S + M_1$ such that

$$sM_1 = \mathrm{i}\Delta S. \tag{2.12}$$

In the expressions above $\Delta = (-1)^{\epsilon_A+1}\partial^R\partial^R/\partial\Phi^A\partial\Phi^*_A$ is a potentially singular operator that must be regularized. If we choose a regularization that keeps the vector symmetry as a preferential one, we obtain, by using standard procedures, that

$$\Delta S = -\frac{\mathrm{i}}{4\pi}\,\mathrm{Tr}\int \mathrm{d}^2x\, \epsilon^{\mu\nu}\big(c\partial_\mu A_\nu - (c-b)\partial_\mu B_\nu\big) \tag{2.13}$$

where B_μ is given by (2.4) and $\epsilon^{01} = -\epsilon^{10} = 1$. We observe that in the QCD$_2$ limit ($g \to \mathbf{1}$, $b \to 0$) ΔS vanishes identically. Furthermore, we can show that even without that limit, g and b do not belong to the cohomology at ghost number one and so there exists some M_1 that solves (2.12). To see this, we note from (2.10) and (2.11) that

$$\begin{aligned} sg &= \mathrm{i}gb + \mathrm{i}[c, b] \\ sb &= -\mathrm{i}b^2 + \mathrm{i}[c, b]. \end{aligned} \tag{2.14}$$

If we represent g as $g = \exp(\mathrm{i}\Lambda)$, Λ taking values in the $SU(N)$ algebra, we can show that

$$s\Lambda = b + \mathrm{i}[\Lambda, \tfrac{1}{2}b - c] - \tfrac{1}{12}[\Lambda, [\Lambda, b]] + \cdots. \tag{2.15}$$

Expressions (2.13) and (2.15) imply that under the linearized BRST transformations, $s^1\Lambda = b$, $s^1 b = 0$, b and Λ form a doublet and therefore are absent from the cohomology [14, 15]. Actually, one can verify that

$$M_1 = -\frac{\mathrm{i}}{4\pi}\,\mathrm{tr}\int_{\partial M} \mathrm{d}x^2\, \epsilon^{\mu\nu}\partial_\mu g g^{-1} A_\nu - \Gamma[g] \tag{2.16}$$

where

$$\Gamma[g] = \frac{1}{12\pi}\epsilon^{\mu\nu\rho}\,\mathrm{tr}\int_M \mathrm{d}^3x\,\left(g^{-1}\partial_\mu g g^{-1}\partial_\nu g g^{-1}\partial_\rho g\right) \tag{2.17}$$

solves (2.12). $\Gamma(g)$ is the Wess–Zumino functional, defined in a three-dimensional manifold M with boundary ∂M representing two-dimensional Minkowski spacetime [12].

To conclude this section, we would like to comment that, although the quantum theory is not obstructed by gauge anomalies, it presents an anomalous divergence of the chiral current, as expected. Constructing a path integral $Z[J]$ by using $W = S_1 + M_1+$ gauge-fixing terms as the quantum action, we can show that the Batalin–Fradkin theorem that implies the independence of $Z[J]$ with respect to the gauge fixing $g = \mathbf{1} + \mathrm{i}\beta$ (see (2.2)) leads to

$$\left\langle \frac{\delta W}{\delta\beta^a}\right\rangle_{\beta^a=0} = \left\langle \left(D_\mu J^\mu_R\right)^a + \frac{1}{4\pi}\epsilon^{\mu\nu}\partial_\mu A^a_\nu \right\rangle_{\beta^a=0} = 0. \tag{2.18}$$

This is a non-trivial result that comes naturally from the QCD$_2$ extension presented above. Observe that the expected values appearing in (2.18) are calculated within the QCD$_2$ sector.

3. Bosonization and soldering

In this section we will consider some fundamental points related to the bosonization of the Abelian version of the theory described above. The non-Abelian case will be described in the next section. Defining $\psi_{R/L} = P_\pm \psi$, we can write the fermionic sector of the Abelianized version of (2.3) as

$$S_F = \int \mathrm{d}^2x \left(\mathrm{i}\bar{\psi}_R \gamma^\mu \left(\partial_\mu - \mathrm{i}B_\mu\right)\psi_R + \mathrm{i}\bar{\psi}_L \gamma^\mu \left(\partial_\mu - \mathrm{i}A_\mu\right)\psi_L\right) \tag{3.1a}$$

$$\equiv S_F^+\left[\bar{\psi}_R, B_\mu, \psi_R\right] + S_F^-\left[\bar{\psi}_L, A_\mu, \psi_L\right] \tag{3.1b}$$

where $B_\mu = A_\mu - \partial_\mu \Lambda$ is the Abelian limit of (2.5). Standard bosonization techniques enable us to derive the bosonized versions of each of the chiral actions S_F^+ and S_F^-, respectively, as†

$$S_+ = \frac{1}{4\pi} \int \mathrm{d}^2x \left[\partial_+\varphi\partial_-\varphi + 2B_+\partial_-\varphi + aB_+B_-\right] \tag{3.2}$$

and

$$S_- = \frac{1}{4\pi} \int \mathrm{d}^2x \left[\partial_+\rho\partial_-\rho + 2A_-\partial_+\rho + bA_+A_-\right]. \tag{3.3}$$

In the above expressions, a and b are free parameters representing the arbitrariness in the regularization procedure. Observe that each one of these bosonic actions represents the fermionic determinant of the corresponding chirality. As is well known, the complete fermionic determinant corresponding to the complete action (3.1) is not just the product of the two chiral determinants since there are interference terms. As pointed out in [16], the complete determinant comes out if we consider the correct Bose symmetrization in the perturbative calculations. An equivalent calculation of the complete bosonized action can be done by using soldering techniques [10, 11], which will be described in what follows. If we gauge the following global symmetries of the action $S_\pm$:

$$\begin{aligned} \delta\varphi &= \alpha \qquad \delta B_\pm = 0 \\ \delta\rho &= \alpha \qquad \delta A_\pm = 0 \end{aligned} \tag{3.4}$$

by using the Noether procedure we can see that the action

$$S = S_+ + S_- - \int \mathrm{d}^2x \left[E_+\mathcal{J}_- + E_-\mathcal{J}_+ - \frac{1}{2\pi}E_+E_-\right] \tag{3.5}$$

with

$$\begin{aligned} \mathcal{J}_+ &= \frac{1}{2\pi}(\partial_+\varphi + B_+) \\ \mathcal{J}_- &= \frac{1}{2\pi}(\partial_-\rho + A_-) \end{aligned} \tag{3.6}$$

is now invariant under (3.4) with $\alpha = \alpha(x)$ a local parameter, if the transformations of the soldering gauge fields are given by $\delta E_\pm = \partial_\pm\alpha$.

† $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$, $\partial_\pm = \frac{1}{\sqrt{2}}(\partial_0 \pm \partial_1)$, $A^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^1)$.

In (3.5) the interference terms depending in $E_\pm$ are those that solder the φ and ρ sectors of the theory. By eliminating $E_\pm$ with the aid of their equations of motion, we arrive at an effective form of S given by

$$\begin{aligned}S' = \frac{1}{4\pi}\int \mathrm{d}^2x\,\big[&\partial_+(\varphi-\rho)\partial_-(\varphi-\rho) + 2A_+\partial_-(\varphi-\rho)\\ &-2A_-\partial_+(\varphi-\rho) + (a+b-2)A_+A_- - 2\partial_+\Lambda\partial_-(\varphi-\rho)\\ &+\partial_+\Lambda\partial_-\Lambda + \Lambda(a(\partial_-A_+ + \partial_+A_-) - 2\partial_+A_-)\big].\end{aligned} \tag{3.7}$$

As one can observe, in (3.7) φ and ρ combine as the collective field [3] $\Phi = \varphi - \rho$ and at the same time the linear combination $\Xi = \varphi + \rho$ is absent from the theory. If we assume that the bosonization has been done keeping the vector symmetry as a preferential one, we must choose the free parameters as $a = b = 1$. This condition also comes out if we assume that the invariance under the usual $U(1)$ (Maxwell) transformations is not lost. As a consequence of (2.6), $U(1)$ gauge invariance is manifest when $\delta A_\pm = \delta B_\pm = \partial_\pm\eta$ and the other fields remain invariant. This process is the one that keeps unitarity and presents the correct Bose symmetry, when a diagrammatic analysis of the bosonization procedure is performed [16]. We see that (3.7) then reduces to

$$S'' = \frac{1}{4\pi}\int \mathrm{d}^2x\,\big[\partial_+(\Phi-\Lambda)\partial_-(\Phi-\Lambda) - (2\Phi-\Lambda)(\partial_-A_+ - \partial_+A_-)\big]. \tag{3.8}$$

Performing an analysis of the set of gauge transformations that keep (3.8) invariant, we see that the soldering symmetry is trivially satisfied due to the definition of Φ. This action is also invariant under the $U(1)$ gauge transformations, but the chiral symmetry (associated with the parameter ϵ in the Abelian version of (2.6)) is lost, which is not surprising, once the invariances presented by the bosonized action must be related to the quantum symmetries of the corresponding fermionic quantum action. Actually, if we add to S' the counterterm

$$M_1 = \frac{1}{4\pi}\int \mathrm{d}^2x\,[\Lambda(\partial_-A_+ - \partial_+A_-)] \tag{3.9}$$

which is the Abelian limit of (2.16), we see that the total action

$$\begin{aligned}W &= S'' + M_1\\ &= \frac{1}{4\pi}\int \mathrm{d}^2x\,\big[\partial_+(\Phi-\Lambda)\partial_-(\Phi-\Lambda) - 2(\Phi-\Lambda)(\partial_-A_+ - \partial_+A_-)\big]\end{aligned} \tag{3.10}$$

besides soldering and $U(1)$ symmetries, is trivially invariant under the chiral symmetry

$$\delta\Phi = \epsilon \qquad \delta\Lambda = \epsilon \tag{3.11}$$

which now also survives the bosonization and soldering processes. It is interesting to note that not only φ and ρ combine in the collective field Φ [3], but also that Λ and Φ themselves combine in a second collective field $\bar\Phi = \Phi - \Lambda$. In terms of $\bar\Phi$, the bosonized version of the extended QED_2 is identical to the bosonized version of the standard Schwinger model. As expected, the bosonized version of the theory presents a much simpler form than its fermionic counterpart, with the action given by (3.1*a*) added to (3.9).

To conclude this section, we would like to consider some aspects associated with the mapping between the currents that appear in both descriptions of the model. The matter current is defined as the object that couples to the vector gauge field in the quantum action

$$J_\pm \equiv \left.\frac{\delta W}{\delta A_\mp}\right|_{\Lambda=0} \tag{3.12}$$

where the condition $\Lambda = 0$ fixes the unitary gauge in order to return to the original description of theory (QED$_2$ in this case). Thus the fermionic currents are mapped in

$$
\begin{aligned}
J_+^F &= \frac{1}{\sqrt{2}}\bar{\psi}(\gamma_0 + \gamma_1)\psi \to J_+^B = -\frac{1}{2\pi}\partial_+\Phi \\
J_-^F &= \frac{1}{\sqrt{2}}\bar{\psi}(\gamma_0 - \gamma_1)\psi \to J_-^B = \frac{1}{2\pi}\partial_-\Phi.
\end{aligned}
\tag{3.13}
$$

As already commented, the introduction of the compensating field g in the fermionic action made it possible to extract the anomalous divergence of the chiral current. In the non-Abelian version, this has been given in equation (2.18), the Abelian limit of which, in lightcone coordinates can be written as

$$
\langle \partial_\mu J_R^\mu \rangle = \langle \partial_+ J_-^F \rangle = -\frac{1}{2\pi}\langle \partial_- A_+ - \partial_+ A_- \rangle \tag{3.14}
$$

obtained by imposing the independence of the vacuum functional with respect to Λ. The corresponding condition applied to the bosonized action (3.10) gives

$$
\partial_+ J_-^B = -\frac{1}{2\pi}(\partial_- A_+ - \partial_+ A_-) \tag{3.15}
$$

which is consistent with the mapping (3.13). Observe that $\partial_- J_+^B + \partial_+ J_-^B$ vanishes identically, since the vector current is conserved. This is expected, since the regularization was done by choosing the vector symmetry as a preferential one. As a final comment, it is interesting to observe that equation (3.15) is just the equation of motion for the field Φ in the usual bosonized Schwinger model, when one uses the definitions of the bosonic chiral currents. This gives the interpretation that the equation of motion for the scalar field in the bosonized version of the Schwinger model is just the bosonized version of the expectation value of the anomalous divergence of the QED$_2$ axial current.

We will show in the next section that similar features also appear when the corresponding non-Abelian models are considered.

4. Non-Abelian extension

Consider now the non-Abelian action (2.3). We can write out the bosonized actions that correspond to each of the chiral sectors as

$$
\begin{aligned}
S_+ &= \frac{1}{4\pi}\int \mathrm{d}^2x \, \mathrm{tr}\left[\partial_+ u^{-1}\partial_- u - 2\mathrm{i}B_+ u^{-1}\partial_- u + aB_+B_-\right] + \Gamma[u] \\
S_- &= \frac{1}{4\pi}\int \mathrm{d}^2x \, \mathrm{tr}\left[\partial_+ v^{-1}\partial_- v - 2\mathrm{i}A_- v^{-1}\partial_+ v + bA_+A_-\right] - \Gamma[v]
\end{aligned}
\tag{4.1}
$$

where again the functional Γ is defined as in equation (2.17) and u and v are elements of the $SU(N)$ group.

As in the Abelian case, we solder the two chiralities by introducing the soldering fields E_+, E_- and defining a new action

$$
S = S_+[u] + S_-[v] - \int \mathrm{d}^2x \, \mathrm{tr}\left[E_-\mathcal{J}_+ + E_+\mathcal{J}_- + \frac{1}{2\pi}E_+E_-\right] \tag{4.2}
$$

where

$$\mathcal{J}_+ = \frac{1}{2\pi}\left[u\partial_+u^{-1} - \mathrm{i}uB_+u^{-1}\right]$$
$$\mathcal{J}_- = \frac{1}{2\pi}\left[v\partial_-v^{-1} - \mathrm{i}vA_-v^{-1}\right]. \tag{4.3}$$

The action (4.2) is invariant under the transformations

$$\begin{aligned}
\delta A_\mu &= \delta g = 0 \\
\delta u &= wu \\
\delta v &= wv \\
\delta E_\pm &= \partial_\pm w - [E_\pm, w]
\end{aligned} \tag{4.4}$$

since (4.4) imply that

$$\delta\mathcal{J}_\pm = -\frac{1}{2\pi}\partial_\pm w + [w, \mathcal{J}_\pm]. \tag{4.5}$$

Eliminating $E_\pm = -2\pi\mathcal{J}_\pm$ with the aid of their equations of motion and introducing $h = u^{-1}v$, we write (4.2) as

$$\begin{aligned}
S' = \frac{1}{4\pi}\int \mathrm{d}^2x\, \mathrm{tr}\big[&\partial_+h^{-1}\partial_-h - 2\mathrm{i}g^{-1}A_+gh\partial_-h^{-1} + 2g^{-1}\partial_+gh\partial_-h^{-1} \\
&-2\mathrm{i}A_-h^{-1}\partial_+h + (a+b)A_+A_- - \mathrm{i}aA_+g\partial_-g^{-1} + \mathrm{i}a\partial_+gg^{-1}A_- \\
&+a\partial_+g^{-1}\partial_-g + 2g^{-1}A_+ghA_-h^{-1} - 2\mathrm{i}g^{-1}\partial_+ghA_-h^{-1}\big] - \Gamma[h].
\end{aligned} \tag{4.6}$$

Again as in the Abelian model, vector symmetry and unitarity are preserved for $a = b = 1$, which is the choice we are going to assume.

The quantum action will also involve the counterterm M_1 of equation (2.16). If we introduce the new field $G = gh$ we find

$$\begin{aligned}
W = S' + M_1 = \frac{1}{4\pi}\int \mathrm{d}^2x\, \mathrm{tr}\big[&\partial_+G^{-1}\partial_-G - 2\mathrm{i}A_+G\partial_-G^{-1} \\
&-2\mathrm{i}A_-G^{-1}\partial_+G - 2A_+GA_-G^{-1} + 2A_+A_-\big] - \Gamma[G]
\end{aligned} \tag{4.7}$$

that has the same form as the bosonized version of the QCD_2 action. So, we see that the compensating field g also shows up in the bosonized non-Abelian case as a collective field. This of course was only possible due to the perfect matching between S' and M_1.

In order to find the correct mapping among the fermionic and bosonic currents we recall that the non-Abelian version of the definition (3.12) is

$$J^a_\pm = \left.\frac{\delta W}{\delta A^a_\mp}\right|_{g=\mathbf{1}}. \tag{4.8}$$

This corresponds in the fermionic formulation to the trivial non-Abelian version of equation (3.13) and in the bosonized formulation, using the quantum action W of equation (4.7); to

$$J_+ \equiv J^a_+T^a = \frac{1}{2\pi}\left(-\mathrm{i}h^{-1}\partial_+h - h^{-1}A_+h + A_+\right)$$
$$J_- \equiv J^a_-T^a = \frac{1}{2\pi}\left(-\mathrm{i}h\partial_-h^{-1} - hA_-h^{-1} + A_-\right). \tag{4.9}$$

Note that, differently from the fermionic case, the current involves the gauge field itself, which is a consequence of the self-interaction presented by the non-Abelian model.

Now, in order to extract the result corresponding to (2.18), we choose

$$G = (\mathbf{1} + \mathrm{i}\beta)h \tag{4.10}$$

with $\beta = \beta^a T^a$ small. At this gauge we find

$$\frac{\delta}{\delta\beta^a} W\bigg|_{\beta=0} = (D_+ J_-)^a - \frac{1}{2\pi}(\partial_+ A_-^a - \partial_- A_+^a). \tag{4.11}$$

Therefore, independence of the theory with respect to β gives

$$(D_+ J_-) = -\frac{1}{2\pi}(\partial_- A_+ - \partial_+ A_-) \tag{4.12}$$

which is the expected result for the anomalous divergency of the chiral current in QCD$_2$, reproduced here in the bosonized formulation.

5. Conclusions

We have shown in this work how to find a bosonization scheme compatible with the introduction of compensating fields in QCD$_2$. The mapping between the matter currents in the fermionic and bosonic formulations was defined in an unambiguous way, by looking at the coupling to the gauge field. We have seen that the bosonic currents involve a non-trivial gauge-field-dependent contribution that is not present in the fermionic description.

Only by including in the bosonic formulation the counterterm that comes from the master equation at one-loop order in the fermionic description do we have the same set of symmetries in both cases. At this point the fact that this counterterm does not depend on fermionic variables and therefore does not need to be bosonized is crucial. Also only by the inclusion of those quantum corrections to the action can the anomalous divergence of the bosonized chiral current be properly derived from the equations of motion of the compensating fields.

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